Nassau County Interscholastic Mathematics League

Contest # 4 **2002-2003**

Team Problems 35 minutes Calculators allowed

T1. In triangle ABE, points C and D are on $\overline{EB}$, C nearer B, in such a way that $\angle BAC \cong \angle CAD \cong \angle DAE$. If AB:AC:AD:AE = 1:3:4:5, find the ratio of lengths BC:CD:DE.

T2. The numbers 409, 505, and 745 all leave the same remainder when divided by the positive integer n. Find the largest possible value of n.

T3. In this cross-number puzzle, shape indicated at right, fill in the correct answers. On your answer sheet, write the solution as a 3 by 4 matrix as it appears in the grid.

1	**2**	**3**	**4**
5			
6			

Across

1. A perfect cube
5. A perfect square
6. A perfect cube

Down

1. A Fibonacci number
2. All odd digits
3. The number of digits in the square of a googol
4. A product of two odd primes

T4. Perry Hotter's magic sock drawer always contains 3 blue socks, 5 black socks and 4 white socks when he opens the drawer. Each morning when he opens the drawer, he takes two socks at random from the drawer. Find the probability that Perry gets matching socks at least once in the next five days. [round your answer to the nearest thousandth].

T5. Let $f(x) = \dfrac{x-3}{x+1}$. Define a sequence of functions by: $\begin{array}{l} f_1 = f \\ f_n = f \circ f_{n-1} \text{ for } n > 1 \end{array}$. Evaluate $f_{2002}(4)$

T6. The letters a and b stand for decimal digits. The six-digit number 322a4b is divisible by 99. Find the value of a+b.

Answers T1. 3:12:20 T2. 48 T3. $\begin{matrix} 1 & 7 & 2 & 8 \\ 4 & 9 & 0 & 0 \\ 4 & 9 & 1 & 3 \end{matrix}$ T4. 0.817 T5. $\dfrac{1}{5}$ T6. 7